Analysis of Coupled Slots and Coplanar Strips on Dielectric Substrate

JEFFREY B. KNORR, MEMBER, IEEE, AND KLAUS-DIETER KUCHLER, STUDENT MEMBER, IEEE

***Abstract*—A frequency-dependent hybrid-mode analysis of single and coupled slots and coplanar strips is presented. The dispersion characteristic and characteristic impedance of the structures are obtained by applying a Fourier transform technique and evaluating the resulting expressions numerically using the method of moments. Numerical results are presented and compared with results published by other investigators. The experimental performance of a slot-line coupler is compared with predicted performance based upon the results presented here for coupled slots. Excellent agreement has been obtained in all cases.**

I. INTRODUCTION

COPLANAR transmission lines have been studied by many investigators, mainly because they are easily adaptable to shunt-element connections without the need to penetrate the dielectric substrate as in the case of microstrip lines. Cohn [1] investigated the slot line and found an approximate analytic expression for the dispersion characteristic and the characteristic impedance by converting the slot line into a rectangular waveguide configuration. Recently this transmission line was analyzed by a new method proposed by Itoh and Mittra [2], but only to the extent that the dispersion characteristics were computed. To the authors' knowledge, there has been no other analysis published for the characteristic impedance of slot line besides Cohn's method [1], [3].

In connection with the increased interest in coplanar transmission lines, the need for an analysis of coupled slot lines or coplanar waveguide (CPW) structures is obvious. Wen [4] studied this transmission line with the assumption that the dielectric substrate is thick enough to be considered infinite for conformal mapping purposes. He also shows some theoretical results for two coplanar parallel strips, again on an infinitely thick dielectric substrate. For large values of ϵ_r, the relative permittivity of the substrate, this assumption may be practical, but it appears impractical for relatively small values of ϵ_r and for thin substrates. An alteration of this method used by Wen is given by Davis *et al.* [5] and takes the finite thickness of the dielectric substrate into account but also uses a quasi-static approximation, and thus lacks any frequency-dependent information on the behavior of phase velocity and characteristic impedance.

The purpose of this paper is to outline a new approach which was first suggested by Itoh and Mittra [2] and then extended by the authors to yield the characteristic impedance of the slot line as well as the dispersion characteristic and characteristic impedances of a pair of coupled slot lines in the odd and the even modes. During the development of the mathematical formulation, it was an easy extension to derive also the characteristics of a pair of coplanar parallel strips. The method is quite general and has also been used to analyze microstrip, although the results will not be presented here.

II. DISPERSION RELATIONSHIP ANALYSIS FOR A SINGLE SLOT LINE

A wave propagation problem on a slot transmission line, shown in Fig. 1, means, in general, the solution to the wave equation in an inhomogeneous medium with inhomogeneous boundary conditions. Moreover, the electric field in the slot is not known, and rather than finding the Green's function in a closed form, the investigator is forced to find an approximate solution. This led Cohn to his approach of using the infinite orthogonal set of waveguide modes, in other words, a complete set of functions, and a conversion from the slot-line configuration into a waveguide configuration by the use of electric and magnetic walls.

Itoh and Mittra [2] introduced a new technique for the analysis of the slot-line dispersion characteristic. To obtain a full understanding of the methods used in this paper, some reiterations from [2] are necessary.

It is known that all hybrid-field components can be obtained from a superposition of TE and TM modes which are related to the two scalar potential functions $\phi^e(x,y)$ and $\phi^h(x,y)$, where the superscripts e and h denote electric and magnetic, respectively. The axial components of TM and TE modes are then

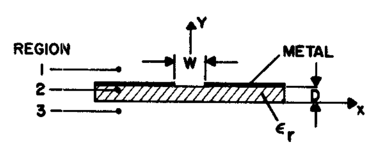


Fig. 1. Slot line.

Manuscript received October 1, 1974; revised February 19, 1975. This work was supported in part by the Office of Naval Research through the U. S. Naval Postgraduate School Foundation Research Program.

J. B. Knorr is with the Department of Electrical Engineering, U. S. Naval Postgraduate School, Monterey, Calif. 93940.

K.-D. Kuchler is with the Federal German Navy. He is currently assigned to the U. S. Naval Postgraduate School, Monterey, Calif. 93940.

$$E_z = k_c^2 \phi^e(x,y) \exp(\pm j\beta z)$$

and

$$H_z = k_c^2 \phi^h(x,y) \exp(\pm j\beta z)$$

respectively, where β is the propagation constant, assuming no losses, and

$$k_c^2 = k_i^2 - \beta^2$$

with

$$k_i = \omega(\epsilon_i \mu_i)^{1/2}, \qquad i = 1,2,3$$

defining the spatial region as shown in Fig. 1.

Both scalar potential functions satisfy the Helmholtz equation which is transformed into the Fourier domain thus converting a second-order partial differential equation into an ordinary differential equation. The solutions to these two ordinary differential equations can then be written as

$$\begin{aligned} \Phi_1^e(\alpha,y) &= A^e(\alpha) \exp[-\gamma_1(y - D)] \\ \Phi_2^e(\alpha,y) &= B^e(\alpha) \sinh \gamma_2 y + C^e(\alpha) \cosh \gamma_2 y \qquad (1) \\ \Phi_3^e(\alpha,y) &= D^e(\alpha) \exp(\gamma_1 y) \end{aligned}$$

$$\begin{aligned} \Phi_1^h(\alpha,y) &= A^h(\alpha) \exp[-\gamma_1(y - D)] \\ \Phi_2^h(\alpha,y) &= B^h(\alpha) \sinh \gamma_2 y + C^h(\alpha) \cosh \gamma_2 y \qquad (2) \\ \Phi_3^h(\alpha,y) &= D^h(\alpha) \exp(\gamma_1 y) \end{aligned}$$

where

$$\gamma_i^2 = \alpha^2 + \beta^2 - k_i^2 \qquad (3)$$

and the subscript defines the region. The preceding equations are found in [2, eqs. (2) and (3)]. It is important to observe that in region 2, $\gamma_2^2 < 0$ for small values of α, which means that the hyperbolic functions are replaced by trigonometric functions.

The eight unknown coefficients A^e through D^h are related to the horizontal electric- and magnetic-field components at the interfaces $y = 0$ and $y = D$ by the continuity conditions, and can be related also to the surface current density on the metal and the electric field in the slot at $y = D$.

If we denote the Fourier transforms of the x- and z-directed current-density and electric-field components by

$$\mathcal{E}_x(\alpha) = \mathfrak{F}\{E_x(x)\} \qquad \mathcal{E}_z(\alpha) = \mathfrak{F}\{E_z(x)\}$$

$$\mathcal{J}_x(\alpha) = \mathfrak{F}\{j_x(x)\} \qquad \mathcal{J}_z(\alpha) = \mathfrak{F}\{j_z(x)\}$$

we obtain a set of coupled equations of the form

$$\begin{bmatrix} M_1(\alpha,\beta) & M_2(\alpha,\beta) \\ M_3(\alpha,\beta) & M_4(\alpha,\beta) \end{bmatrix} \begin{bmatrix} \mathcal{J}_x(\alpha) \\ \mathcal{J}_z(\alpha) \end{bmatrix} = \begin{bmatrix} \mathcal{E}_x(\alpha) \\ \mathcal{E}_z(\alpha) \end{bmatrix} \qquad (4)$$

where the elements of the M-matrix are the Fourier transforms of dyadic Green's function components.

If the M-matrix is inverted, we obtain a new matrix N and a second set of coupled equations

$$\begin{bmatrix} N_1(\alpha,\beta) & N_2(\alpha,\beta) \\ N_3(\alpha,\beta) & N_4(\alpha,\beta) \end{bmatrix} \begin{bmatrix} \mathcal{E}_x(\alpha) \\ \mathcal{E}_z(\alpha) \end{bmatrix} = \begin{bmatrix} \mathcal{J}_x(\alpha) \\ \mathcal{J}_z(\alpha) \end{bmatrix}. \qquad (5)$$

This last formulation is equivalent to [2, eq. (4)].

Up to this point, the formulation of the problem is exact; no approximation has been made. If, however, the electric-field and current-density components are expanded in infinite series using a complete set of basis functions, and Galerkin's method [6] is applied, a homogeneous system of linear equations can be found [2, eqs. (7) and (8)]. An iteration scheme for β can be used to find a nontrivial solution for this set of equations.

The remaining question is what kind of basis functions to choose. The choice of this complete set of basis functions is arbitrary in a mathematical sense, since as long as this set is complete, any closed form of the field components can be represented by it. However, the rate of convergence of the series representation will depend on how well the first few terms approximate the closed form. In general, this requires some *a priori* knowledge of the true distributions.

In order to determine the sensitivity of the previously outlined method to the choice of basis functions, an investigation of various one-term approximations was made. The electric-field components were assumed to be of the form

$$e_x = \begin{cases} 1, & |x| < W/2 \\ 0, & \text{elsewhere} \end{cases} \qquad (6a)$$

$$e_z = \begin{cases} -1, & -W/2 < x < 0 \\ 1, & 0 < x < W/2 \\ 0, & \text{elsewhere.} \end{cases} \qquad (6b)$$

Another choice which certainly approximates the fields more closely is

$$e_x = \begin{cases} [(W/2)^2 - x^2]^{-1/2}, & |x| \leq W/2 \\ 0, & \text{elsewhere} \end{cases} \qquad (7a)$$

$$e_z = \begin{cases} x[(W/2)^2 - x^2]^{1/2}, & |x| \leq W/2 \\ 0, & \text{elsewhere} \end{cases} \qquad (7b)$$

which was used in [2].

The use of x- and z-directed field components is called the second-order approximation. A reduction of computer time is possible by assuming $E_z = 0$, which we will refer to as the first-order approximation. The problem then reduces to the evaluation of a single integral instead of four during each iteration for β. In Fig. 2, the results from these four different approximations are shown, namely the first- and second-order approximations with either basis-function set (6) or (7), and are compared with results from [3]. Although no indication about the rate of convergence and hence the accuracy of the two series

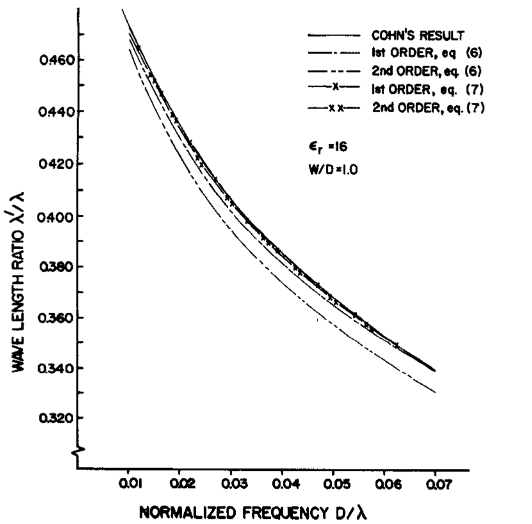


Fig. 2. Dispersion characteristic of a single slot.

for a general problem can be obtained, a comparison for this particular problem shows that the largest deviation between the different approximations is less than 4 percent. It is to be noted that the basis set given by (7) is superior to that given by (6) and that the first- and second-order solutions based on (7) agree very well with Cohn's results. The first- and second-order solutions based upon (6) give less accurate results with the second-order approximation being the poorer of the two due to the physically impossible discontinuities of (6b).

Using one specific set of parameters in the 1–3-GHz range, a comparison of the magnitudes of the x and z components of electric field was made. The x-directed electric-field component was found to have a magnitude greater than ten times that of the z-directed component. This provides further justification for use of the more efficient first-order approximation ($E_z = 0$).

III. CHARACTERISTIC IMPEDANCE OF A SINGLE SLOT LINE

The definition of the characteristic impedance for an ideal transmission line is uniquely defined by static quantities, but is somewhat arbitrary for the slot transmission line due to the non-TEM nature of this problem. One possible choice is to define it as

$$Z_0 = \frac{V_0^2}{2P_{\text{avg}}} \tag{8}$$

where V_0 is the slot voltage and P_{avg} is the time-averaged power flow on the slot line which is given by

$$P_{\text{avg}} = \tfrac{1}{2}\,\text{Re} \iint_s \boldsymbol{E} \times \boldsymbol{H}^* \cdot \boldsymbol{a}_z \, dx\, dy. \tag{9}$$

The field components in this integral can be expressed in terms of the scalar potential functions by the use of Maxwell's equations.

Since the slot line is an open-boundary structure, the limits of integration in (9) are infinite, which makes the use of Paseval's theorem feasible. By this method (9) is transformed into the spectral domain where again a double integral is obtained with the variables of integration α and y instead of x and y. Equation (9) is then of the form

$$P_{\text{avg}} = \frac{1}{4\pi}\,\text{Re} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[\Phi^e(\alpha),\, \Phi^h(\alpha),\, \frac{\partial \Phi^e(\alpha)}{\partial y},\, \frac{\partial \Phi^h(\alpha)}{\partial y},\, \beta,\, \alpha,\, y\right] d\alpha\, dy. \tag{10}$$

The integral in (10) is a function of the Fourier transformed scalar potentials which were given in (1) and (2). Since the dispersion problem was already solved and the dependence of the coefficients $A^e(\alpha)$ through $D^h(\alpha)$ on the electric-field distributions is known, the integral of (10) can be evaluated.

Equations (1) and (2) show a simple functional dependence on the variable y, and thus integration with respect to y may be accomplished analytically in (10). One then obtains a single integral of the form

$$P_{\text{avg}} = \frac{1}{4\pi}\,\text{Re} \int_{-\infty}^{\infty} g(\alpha,\beta)\, d\alpha. \tag{11}$$

This integral has to be separately evaluated for the three spatial regions since the solution to the wave equation differs in each of these, as does the integrand of (11). Finally, to get all the necessary coefficients for (8), the voltage V_0 has to be computed. This involves simply the integration of the assumed electric-field distribution across the slot and can be done analytically. The evaluation of (11) must be done numerically on a digital computer. Although the limits of integration are infinite, the rapid decay of the integrand for large values of α and its well-behaved form make this computational task routine.

It should be noted that the amount of algebraic complexity in (11) is quite large and lengthy, and for this reason, the details are not presented here. For numerical purposes, the complexity is somewhat reduced by neglecting the z-directed electric-field component which means that the spatial phase shift between E_x and E_z requires no algebraic manipulation. It will be shown in the comparison between this and Cohn's method that use of the first-order solution results in good agreement. A computer program was developed which first computes the dispersion characteristic and then the characteristic impedance. Although the first- and second-order approximations for the dispersion calculation, as well as the two distributions (6) and (7) were investigated, the following results were computed using a second-order solution for the dispersion calculations [see (7)] and a first-order solution for characteristic-impedance calculation.

The comparison of these results with the values obtained by Mariani *et al.* [3] is shown in Fig. 3 and indicates a very close agreement for the two arbitrarily chosen sets of parameters. The average computation time on the IBM

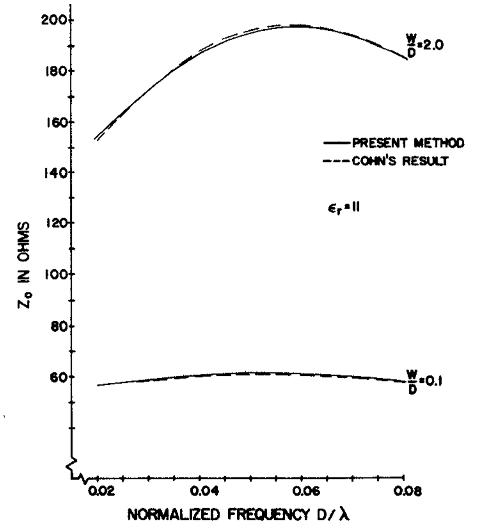


Fig. 3. Characteristic impedance of a single slot.

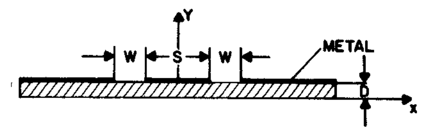


Fig. 4. Coupled slots.

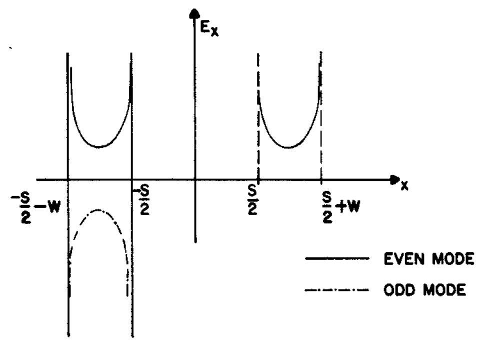


Fig. 5. Assumed field distribution for coupled slots.

360 computer was 27 s for each value of D/λ. A first-order solution for the dispersion characteristic used in connection with distribution (6) required only 2.5 s and produced results which differed by less than 10 percent from those obtained using the second-order solution.

IV. CHARACTERISTICS OF A PAIR OF COUPLED SLOT LINES OF EQUAL WIDTH

Since the mathematical method developed so far produced results in agreement with those obtained by Cohn (which have been confirmed by several experiments) the method was applied to the analysis of two coupled slot lines, the geometry of which is shown in Fig. 4. This extension is straightforward for the following reason. The coefficients N_1 through N_4 are functions of α, β, ϵ_r, and D, and are independent of the slot configuration. The slot configuration enters into the calculation only through coefficients in the assumed field distributions or basis functions. Thus it is necessary only to modify (7) such that the mathematical description of the basis functions corresponds to the physical configuration and field distributions of the coupled slots.

Fig. 5 shows the two assumed distributions of electric field e_x for the even and odd modes, respectively. The change in the field component e_z is similar. In the Fourier domain, one simply applies the shifting theorem to the transforms of (7) to obtain the new transforms. Furthermore, (8) is changed to

$$Z_0 = \frac{V_0^2}{P_{\text{avg}}} \tag{12}$$

since the total time-averaged power surrounding the transmission lines is now due to two lines. Beyond this change, no major modifications were necessary to use the existing computer program. Figs. 6 and 7 show the results for several values of ϵ_r where Z_{0e} and Z_{0o} denote the even- and odd-mode characteristic impedances, respectively. A first-order approximation was used to calculate the dispersion and the characteristic impedances to reduce the

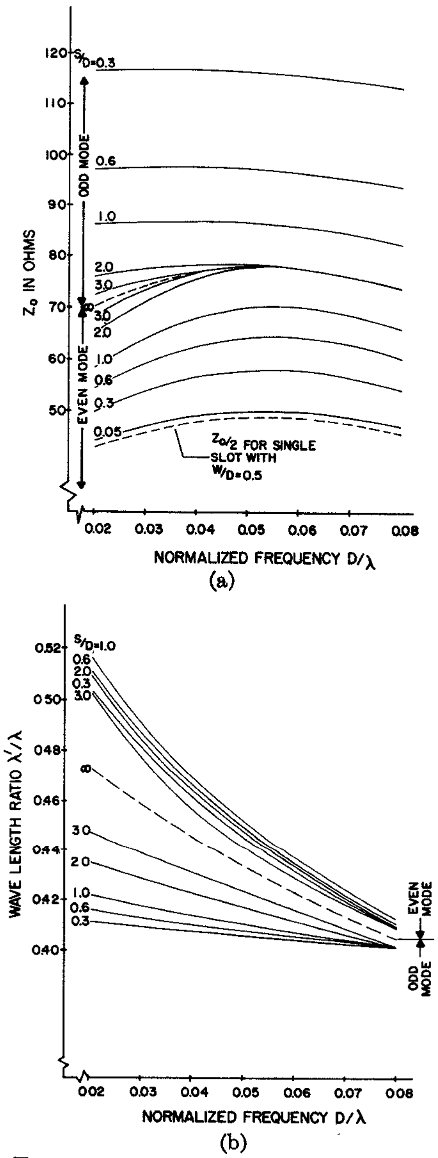
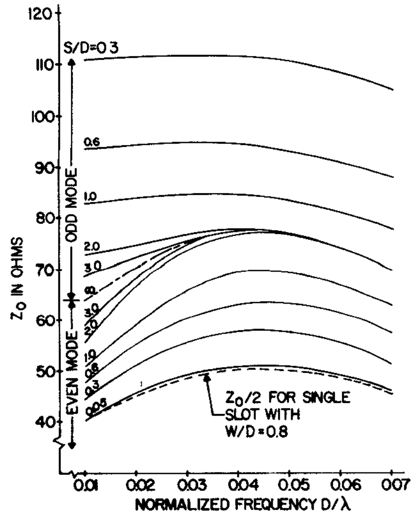
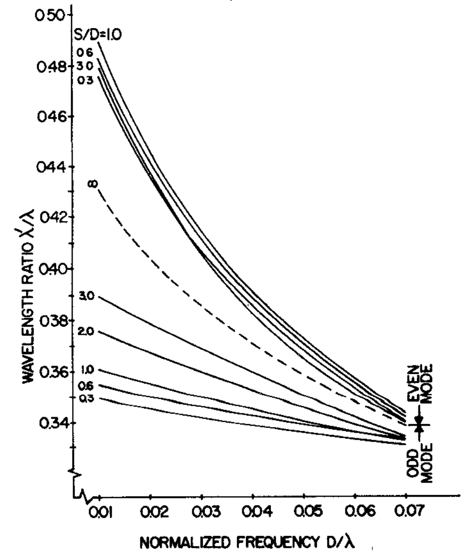


Fig. 6. (a) Even- and odd-mode characteristic impedances for coupled slots with $W/D = 0.25$, $\epsilon_r = 11$. (b) Even- and odd-mode dispersion characteristics for coupled slots with $W/D = 0.25$, $\epsilon_r = 11$.

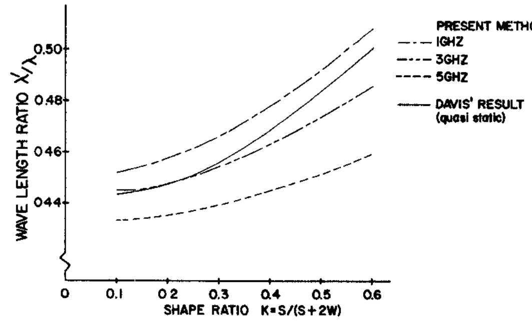


(a)

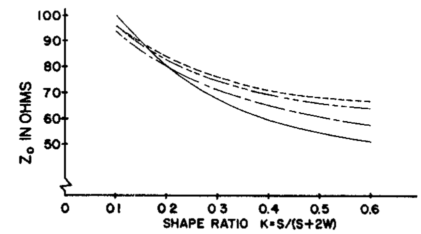


(b)

Fig. 7. (a) Even- and odd-mode characteristic impedances for coupled slots with $W/D = 0.4$, $\epsilon_r = 16$. (b) Even- and odd-mode dispersion characteristics for coupled slots with $W/D = 0.4$, $\epsilon_r = 16$.



(a)



(b)

Fig. 8. (a) Dispersion characteristic and (b) characteristic impedance for CPW with $W/D = 1$, $\epsilon_r = 10$.

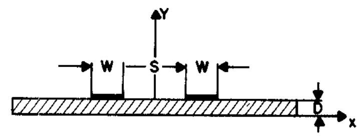


Fig. 9. Coplanar strip transmission line.

computer time which was, on the average, 80 s for both the odd and even modes. To the authors' knowledge, these results are basically new in the literature and can be only partially compared to other results. A quantitative comparison between these results for the odd-mode characteristic impedance and the quasi-static CPW impedance calculated by Davis *et al.* [5] can be made and is shown in Fig. 8 for $W/D = 1$ (or in the notation of [5], $t = s$) at frequencies of 1, 3, and 5 GHz. Reasonable agreement exists for the lower frequency range. Note that the characteristic impedance of the CPW is one half of Z_{0o} for equal slot widths. Another qualitative check on these results can be made by investigating the even-mode characteristic impedance in the limiting case as S, the separation between slots, becomes very small. One expects that Z_{0e} will be close to one half of Z_0, where Z_0 is the characteristic impedance of a single slot whose width is twice the width of the slot in the coupled structure. The dispersion characteristic for both structures, however, should be the same. This comparison is made in Figs. 6(a) and 7(a). It is interesting to observe the coupling and decoupling between the waves in the two slots as the frequency varies for large values of S/D. As the frequency increases, the waves become more closely bound to the slot which means there is less interaction between the two waves. In this case, Z_{0e} and Z_{0o} converge to Z_0, the characteristic impedance of a single slot with no coupling.

Another interesting phenomenon is the fact that for a fixed D/λ the ratio λ'/λ in the even mode first increases and then decreases as S/D increases from very small to larger values as shown in Figs. 6(b) and 7(b). An explanation may be given as follows. For small enough values of S/D, the metal strip between the slots has little effect on the propagating wave, and the wave propagates as if it were in a slot of width $2(W/D) + S/D$. Increasing the separation between the slots effectively increases slot width (the metal separation still has little effect), and the ratio λ'/λ increases. As S/D continues to increase, the two waves start to decouple and behave more as two waves on two slot lines which will finally be totally decoupled. Each wave then propagates on a slot line with width W/D, hence λ'/λ decreases.

V. CHARACTERISTICS OF COPLANAR STRIP LINE

A configuration of a pair of coplanar strip lines is shown in Fig. 9. The dispersion characteristic and the characteristic impedance can be found by again using Galerkin's method in the Fourier transform domain. Since an ap-

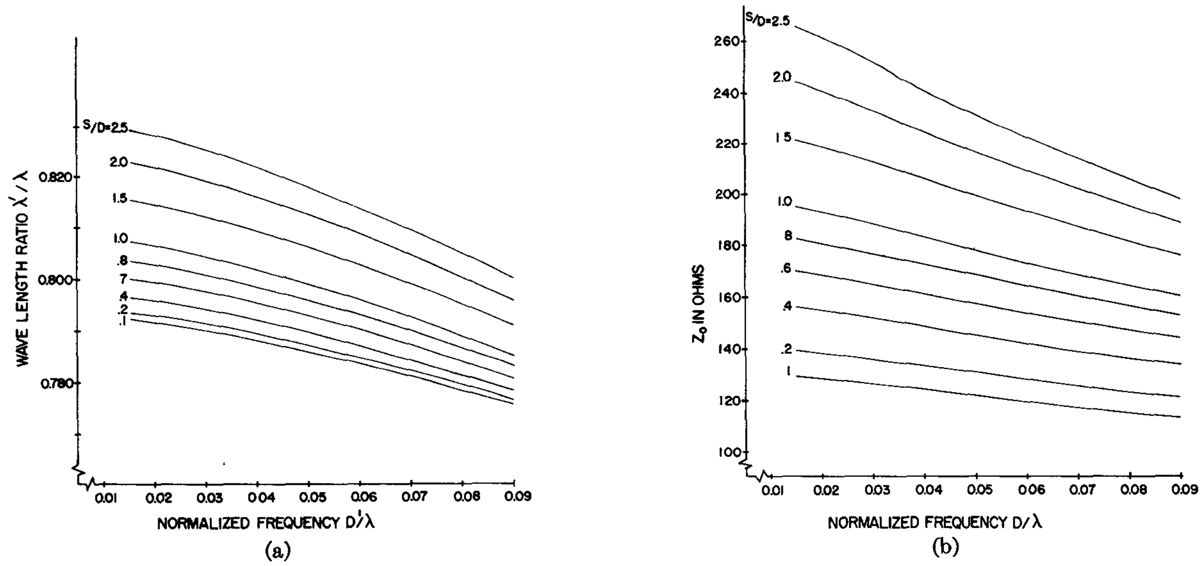


Fig. 10. (a) Dispersion characteristics of coplanar strips with $W/D = 1.5$, $\epsilon_r = 2.5$. (b) Characteristic impedance of coplanar strips with $W/D = 1.5$, $\epsilon_r = 2.5$.

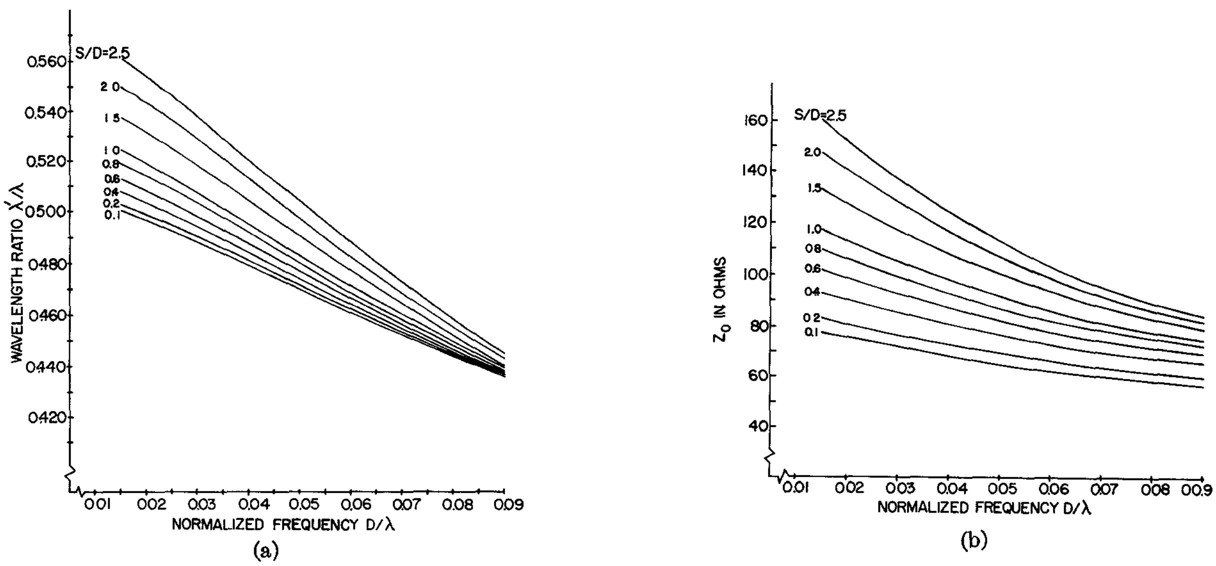


Fig. 11. (a) Dispersion characteristics of coplanar strips with $W/D = 1.5$, $\epsilon_r = 9$. (b) Characteristic impedance of coplanar strips with $W/D = 1.5$, $\epsilon_r = 9$.

proximation of the current density across each strip is more feasible than an approximation of the electric field at $y = D$, the equation set (4) was used to determine the dispersion characteristic. A first-order solution was obtained assuming that the surface current in the x direction was negligible and that the z-directed surface current was of the form

$$j_z(x) = \begin{cases} \dfrac{\pm 1}{\{(W/2)^2 - [x \pm (S+W)/2]^2\}^{1/2}}, & S/2 < |x| < S/2 + W \\ 0, & \text{elsewhere} \end{cases} \qquad (13)$$

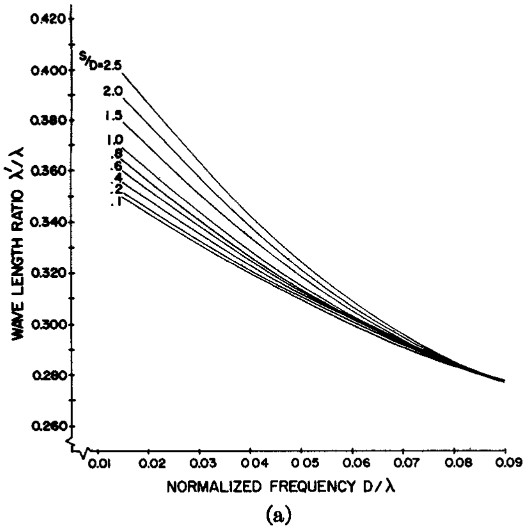
over each strip. The characteristic impedance was calculated as

$$Z_0 = 2P_{avg}/I_0^2 \qquad (14)$$

where I_0 is the total current on one strip. Any further necessary formulations were very similar to the previously outlined procedure for slot lines. Three representative graphs for the dispersion characteristics and characteristic impedances are shown in Figs. 10–12. Reasonable agreement for the impedances is found by comparing the present values with the results by Wen [4]. However, as one might expect, the present method yields somewhat larger values for the impedance due to the finite dielectric substrate.

VI. EXPERIMENTAL RESULTS

Although all previous comparisons showed reasonable agreement with existing results, some effort was devoted to obtaining experimental verification of this work. One



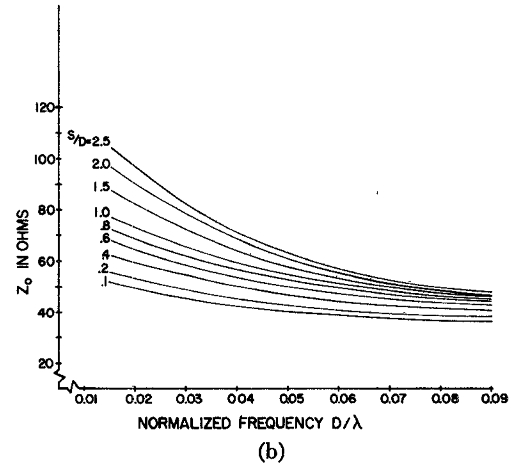


Fig. 12. (a) Dispersion characteristics of coplanar strips with $W/D = 1.5$, $\epsilon_r = 20$. (b) Characteristic impedance of coplanar strips with $W/D = 1.5$, $\epsilon_r = 20$.

experimental check is provided in the work of Luna [7] who measured the characteristics of coplanar strips and found an agreement of better than 5 percent between theory and experiment. The authors further confirmed the accuracy of the results for coupled slots by constructing a slot-line coupler.

Coupled slots ($W/D = 0.470$, $S/D = 1.08$) were etched onto one side of a 1-oz copper surface on a dielectric substrate with $D = 0.125$ in, and $\epsilon_r = 16$. Microstrip-to-slot transitions [8] were used at three ports, and the fourth was terminated with a chip resistor. A center frequency of 3 GHz ($l = 1.6$ cm) was chosen.

The work of Jones and Bolljahn [11] has been extended by Zysman and Johnson [9] who have derived the impedance matrix of dispersive coupled lines. Using the results presented earlier, the elements of the impedance matrix of the coupler were evaluated, and its performance as a function of frequency was calculated. The theoretical performance is compared with experiment in Fig. 13. Good correlation is evident. The deterioration of directivity noted experimentally at the band edges is in all probability due to the rising VSWR of the microstrip–slot transitions at these frequencies. It should be noted that the behavior of this coupler is different than that of the contradirectional (nondispersive) strip-line coupler. Here the difference in the phase velocities of the odd and even modes results in codirectional coupling as in waveguide (see also Mariani and Agrios [10]). Further investigations into the behavior of dispersive couplers have been undertaken and preliminary results have been reported elsewhere [12].

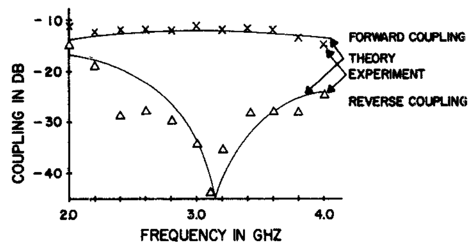


Fig. 13. Theoretical and experimental response for a slot-line coupler.

VII. CONCLUSIONS

An efficient numerical method has been presented for obtaining the dispersion characteristics and the characteristic impedances for a single slot line, two parallel slot lines of equal width, and two parallel coplanar strips. Solutions to the wave propagation problem were obtained in the Fourier transform domain. Numerical results obtained by this method have been presented and compared or related to other existing data and to experiment. In all cases, good agreement was obtained.

The transform technique is relatively straightforward in concept, but extensive algebraic manipulation of the resulting equations is required to achieve computational efficiency. The labor involved should not be underestimated. For this reason, the equations have not been presented here in detail. The authors expect to make this information available in a technical report in the near future. It is also anticipated that design curves will be made available in a technical report.

ACKNOWLEDGMENT

The authors wish to acknowledge the support provided by the Naval Postgraduate School Foundation Research Program and the W. R. Church Computer Center, also of the Naval Postgraduate School.

REFERENCES

[1] S. B. Cohn, "Slot line on a dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.

[2] T. Itoh and R. Mittra, "Dispersion characteristics of slot lines," *Electron. Lett.*, vol. 7, pp. 364–365, July 1971.

[3] E. A. Mariani, C. P. Heinzman, J. P. Agrios, and S. B. Cohn, "Slot line characteristics," *IEEE Trans. Microwave Theory Tech.* (*1969 Symp. Issue*), vol. MTT-17, pp. 1091–1096, Dec. 1969.

[4] C. P. Wen, "Coplanar waveguide: A surface strip transmission line suitable for nonreciprocal gyromagnetic device applica-

tions," *IEEE Trans. Microwave Theory Tech.* (*1969 Symp. Issue*), vol. MTT-17, pp. 1087–1090, Dec. 1969.
[5] M. E. Davis, E. W. Williams, and A. C. Celestini, "Finite-boundary corrections to the coplanar waveguide analysis," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-21, pp. 594–596, Sept. 1973.
[6] D. S. Jones, *The Theory of Electromagnetism.* New York: Pergamon, 1964.
[7] A. E. Luna, "Parallel coplanar strips on a dielectric substrate," M.E.E. thesis, Dep. Eng., Naval Postgraduate School, Monterey, Calif., 1973.
[8] J. B. Knorr, "Slot-line transitions," *IEEE Trans. Microwave Theory Tech.* (Short Papers), vol. MTT-22, pp. 548–554, May 1974.
[9] G. I. Zysman and A. K. Johnson, "Coupled transmission line networks in an inhomogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 753–759, Oct. 1969.
[10] E. A. Mariani and J. P. Agrios, "Slot-line filters and couplers," *IEEE Trans. Microwave Theory Tech.* (*1970 Symp. Issue*), vol. MTT-18, pp. 1089–1095, Dec. 1970.
[11] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 75–81, Apr. 1956.
[12] K.-D. Kuchler and J. B. Knorr, "Coupler design using dispersive transmission lines," Proc. 8th Asilomar Conf. Circuits, Systems and Computers (Pacific Grove, Calif.), Dec. 3–5, 1974.

Scattering Matrices of Junction Circulator with Chebyshev Characteristics

JOSEPH HELSZAJN, MEMBER, IEEE

***Abstract*—The purpose of this paper is to derive the scattering matrix of junction circulators with Chebyshev characteristics. This is done by forming the overall eigenvalues of the circulator one at a time in terms of the *ABCD* matrix of the matching network and the initial set of the junction eigenvalues. This paper deals both with the case where the frequency variation of the in-phase eigennetwork at the gyrator terminals is neglected compared to that of the counterrotating ones, and with the case where it is included. It is found that the former approach is in excellent agreement with the results obtained by assuming a 1-port model for the circulator. The influence of this eigennetwork on the overall frequency response is studied separately by combining the electromagnetic and network problems in the case of the stripline circulator.**

INTRODUCTION

THE THEORY of wide-band circulators using external matching networks usually starts by assuming that the equivalent circuit of the device is a 1-port network [1]–[8]. This 1-port circuit consists of a shunt conductance in parallel with either a lumped or distributed resonator. It assumes that the frequency behavior of the in-phase eigennetwork at the gyrator terminals may be omitted compared with that of the two counterrotating ones. The bandwidth over which this approximation applies has been discussed in [8] in terms of the resonant frequencies of the counterrotating eigennetworks, but a fuller investigation of the omission of the frequency variation of the in-phase eigennetwork on the quality of this equivalent circuit appears desirable.

The most general representation of the 3-port circulator is in terms of the eigenvalues of the scattering matrix [9]. The eigenvalues of this matrix are reflection coefficients associated with the different ways of exciting the junction. The entries of the scattering matrix are constructed by taking linear combinations of these eigenvalues. This method therefore yields not only the reflection coefficient at the input port but also the transmission coefficients of the junction. The approach is quite general and applies to the m-port junction also. It starts by representing the matching network at each port by its *ABCD* matrix. The eigenvalues at the input terminals of the junction are then obtained one at a time in terms of the *ABCD* parameters and the initial set of eigenvalues at the gyrator terminals.

In this paper the boundary condition for circulators with Chebyshev frequency characteristics is first established at the terminals of the matching network in terms of the eigenvalues of the scattering matrix by omitting the frequency variation of the in-phase eigennetwork at the gyrator terminals and subsequently reintroducing it to study its influence on the overall frequency response. It is found that the former results are in excellent agreement with those obtained by connecting the matching network directly to the 1-port circuit [8].

The influence of the in-phase eigenvalue on the overall frequency response of the circulator is studied separately in the case of the stripline circulator by combining the

Manuscript received July 5, 1974; revised March 4, 1975.
The author is with the Department of Electrical Engineering, Heriot-Watt University, Edinburgh EH1 2HT, Scotland, U.K.